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Independent Component Analysis and an Application in The Field of Health

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Abstract

Independent Component Analysis has been used for 10 years in the fields of statistics and neuorophysiology. Therefore, ICA is completely new and original technique. It is established on the basis of Factor Analysis. ICA is used for decomposing the components in order to obtain clear and analyzable subcomponents. Method seems to be useful particularly in deartifacting procedure.

In this research an overview of ICA has been tried to made. EEG is used as an example. Main purpose of this study is to provide a basic reference to the researchers about the ICA on which there is no enough work in Turkey. Besides, it is attempted to show that it is a good method which can be used in several areas of health sciences. It is hoped that exploration of the mathematical and statistical basis of ICA could illuminate to the Turkish scientists who might interested in ICA. This research is doctorate thesis of author.

Keywords: Independent Component Analysis (ICA), Bayesian Theory, Main Component Solution, Factor Solution.

1. INTRODUCTION

Independent Component Analysis (ICA-) is one of the methods of maximizing the statistical independence between variables. It can be seen as an extension of factor analysis and principal component analysis. Although its basic assumption is non-gaussanity; Before starting the analysis, the data structure undergoes many transformations and simplifications. The most commonly used criteria are; to find the degree of uncertainty by using multivariate statistical independence theories and information theory. In addition, depending on the developments in recent years, it has become a probabilistic model in which time series are also used. The analysis, which turns into an optimization problem by making use of maximum likelihood and Bayesian approaches, uses almost the entire mathematical infrastructure of multivariate statistical methods (17,18).

As the variables measured in health researches increase, depending on the developments in their control, in the process; monitoring and diagnosis is gaining importance. The handling or elimination of these variables is a major problem due to rapid detection, size of the error, corruption of the process, and other special circumstances. With this; again, hundreds or thousands of records are kept every day for hundreds of variables handled in a single processing unit. If we do not have a special discrimination method, the information we can obtain from such data will be very limited. Because of this reason; will be able to determine the properties of the data with key variables and directly adapt it to a small size visualization; We will need a high-level processing tool. For a high-dimensional analysis and separation of a dependent data set; There is a need for tracking schemes that can be constructed with multivariate methods adapted from "principal component analysis" and "partial least squares" methods(17,18).

Although the ICA is seen as an extension of Principal Component Analysis, its purpose is different. Principal Component Analysis is a size reduction method that transforms data into a smaller, uncorrelated data set, so that most of the original variance is retained in the model. However, simply making the variables uncorrelated does not make them independent, that is, in Principal component analysis; While orthogonalization the direction vectors of second order statistics (mean and variance) methods are used and independence is enforced; There is no obligation to orthogonolİzing in ICA and includes high-order statistics. Only the second level does not establish statistical independence. At the same time, only a high level can statistically reduce dependency. Therefore, the individual components provide more useful information from the observed data than the principal components(17,18).

The aim of this study is to deal with the mathematical background of ICA and Bayesian approaches and to try to show that a very short EEG data can become more understandable with ICA.

2. INDEPENDENT COMPONENT ANALYSIS –ICA

Independent Component Analysis (ICA) is a statistical method used to determine subgroups or hidden factors that can be formed by random variables obtained from measurements and signals (9,10,11).

BBÇ; can reveal large databases by generalizing them with a multivariate model. In this model, some hidden variables in an unknown complex system may be linear or nonlinear or a mixture of both. The feature of this method is that latent variables may not show normal distribution or be mutually independent. For this reason, they are called "independent components" of the observed data. These variables detected by ICA; They are also called factors or resources. ICA; It can be seen as an extension of principal component analysis and factor analysis methods. These classical methods are also used to detect factors, but they fail to reveal the underlying hidden conditions. ICA is a more powerful technique than these methods (9,10,11).

Simplifying and easily interpreting a multivariate data; It is a common problem of many disciplines such as statistics, signal processing, neural network research. We see that the job of presenting the structure of the original data in a simple way is most often done with linear transformations. The most well-known methods involving linear transformations are principal component analysis, factor analysis, “projection pursuit” methods, as mentioned above. Like all these methods, ICA tries to minimize the statistical dependence between variables and components (9,10,11).

As is known in principal component analysis, a multivariate data set; It is transformed into a new collection of components in which the correlation between them is minimal but the variables within itself have the maximum correlation. ICA is an extension of this. Only the dependence between the resulting components geometrically selects orthogonal transformation methods. So the addiction disappears. In addition, although the input variables do not show normality, the result variables now fit the normal distribution (9,10,11).

If we represent the observed dataset, where the sources are complex and perceived as a single S source (consisting of unknown sources), with a complex linear matrix A, this matrix is processed in ICA procedures to create an X measurement (signals) vectors, and ultimately aim to transform into statistically independent Y vectors. Since the input variables are dependent and uncertain, they do not show normal distribution. But the resulting variables are still discrete, that is, independent.

Consider a multivariate time series {xi(t)}, i = 1,...,n. It is also defined by a complex process in the $x_i(t) = \sum_{j=1}^{n} a_{ij} s_j(t)$ form of these specified signals. x(t) = [x1(t),x2(t),...,xn(t)] If vector T is defined as observed data and sj (t) as unknown signals.

The matrix B, which is a mixture of sources, can be used as y(t) = Bx(t)= BAs(t). So the problem turns into finding the Unknown mixer matrix A. This matrix consists of a mixture of constant signals over time(9,10,11). It is possible to subtract the number of sensors from the number of

sources. Well; The matrix A need not be a square matrix. For n observation data; If the number of sources and the number of sensors are equal, matrix A is an n×n square matrix.

If we denote it by B = A-1, y(t) = s(t) and the separation process turns into its cleanest form. In practice, optimizing y will be possible using a scaled version of s. So we can just use BA = PD to find B . Here, P is the permutation matrix and D is the diagonal scaling matrix (9,10,11).

Where the signals are independent or in any other possible form; ICA is the easiest way to decode and understand such a large data, as well as in sequential and time-dependent terms compiled from these signals. Many different algorithms are used in the implementation of ICA. These are block-based or direct access (on-line) methods. Block-based algorithms take all the data at once, process it and give results. Direct access algorithms detect each data continuously. The problem that seems to be a disadvantage in these algorithms is the process of selecting a parameter such as learning speed by correcting it. This is particularly evident at the point of detection in the direct access method. If the detection rate is chosen smaller than necessary, the result may be found more slowly or even not found at all. If the detection speed is selected too large. The algorithm can go into an endless loop (blow-up) (9,10,11).

Independent component analysis is a measurable and statistical method used to reveal hidden factors that underlie random variables, measures, and signals.

The BBB defines a generative model for observed multivariate data, given as large databases of models. In the model, the data variables are assumed to be systems of latent mixing and a linear mix of unknown latent variables. These latent variables do not follow the gauss distribution and are assumed to be mutually independent and are called independent components of the observed data. These individual components, also called sources or factors, are found by the ICA.

We see that the hidden resource allocation problem is shortened by finding a linear expression with a statistically dependent component. In tried cases, we cannot find an expression where the components are completely independent, but we do find at least as many independent components as they can be. This gives us the definition of ICA as follows. A series of measurements of random values are given; , $(x_1(t), x_2(t), ..., x_n(t))$ where t represents the generalized sample content as a mixture of independent components or time t.

$$\begin{pmatrix} x_1(t) \\ x_2(t) \\ . \\ . \\ . \\ x_n(t) \end{pmatrix} = A. \begin{pmatrix} s_1(t) \\ s_2(t) \\ . \\ . \\ . \\ s_n(t) \end{pmatrix}$$

Here A is an unknown matrix. When measuring Xi(t), we calculate both matrix A and Si(t) in Independent Component Analysis. Suppose that many independent components Si are equal to many measured values.

In another alternative, we define ICA as follows;

It is to find a linear transition given by matrix W by giving random values to Yi. Here i=1,….,n and n is as independent as possible. This formula is really not much different from the previous formula. When you calculate A, its inverse gives W(9,10,11). The model described by the problem (provided and if the components are non-gaussian) can be calculated. This is an important requirement that explains the main difference between ICA and factor Analysis. In reality, we can calculate ICA as non-gaussian factor Analysis as well as model data that is some linear mixture underlying factors (9,10,11). Since the generalized ICA model has different applications in many areas, we will discuss some parts of it later.

We have many different sources in the brain such that the signals the brain emits are a mix of many sensors outside of our heads. This situation can simply be compared to the hidden resource allocation model (9,10,11).

2.1. DISPLAY FORMAT AND PROPERTIES OF MULTI-DIMENSIONAL VECTORAL STRUCTURES WHICH ARE INTERESTED IN INDEPENDENT COMPONENT ANALYSIS

Cumulative distribution function for x=x0 of the random variable x

$$F_x(x_0) = P(x \le x_0) \qquad -\infty < x_0 < +\infty$$

$$0 \le F_x(x_0) \le 1 \qquad F_x(-\infty) = 0 \qquad F_x(+\infty) = 1$$

The differential of the cumulative distribution function for the continuous random variable x gives the probability density function(9,10,11).

$$P_x(x_0) = \left.\frac{dF_x(x)}{dx}\right|_{x=x_0}$$

The inverse is;

$$F_x(x_0) = \int_{-\infty}^{+\infty} P_x(\varepsilon) d\varepsilon$$

X : being an n-dimensional random vector;

$$X = (x_1, x_2, ..., x_n)^T$$

$$F_x(x_0) = P(x \le x_0)$$

For the multivariate cumulative distribution function (for all components of vector X);

$$P_x(x_0) = \frac{\partial}{\partial x_1}\frac{\partial}{\partial x_2}....\frac{\partial}{\partial x_n}F_x(x)\Big|_{x=x_0}$$

Like this;

$$F_x(x_0) = \int_{-\infty}^{+\infty} P_x(x)dx = \int_{-\infty}^{x_{0,1}}\int_{-\infty}^{x_{0,2}} .... \int_{-\infty}^{x_{0,n}} P_x(x)dx_n ...dx_2 dx_1$$

If i_{th}.component of the vector of x_0 is $x_{0,i}$;

Joint distribution: $Z^T=(X^T,Y^T)$: supervector $F_{x,y}(x_0, y_0) = P(x \le x_0\ , y \le y_0)$

The mean vector: $\mu_x = E\{X\} = \int_{-\infty}^{+\infty} x.P_x(x)dx$

The i_{th} component of the mean vector is: $\mu_{x_i} = E\{X_i\} = \int_{-\infty}^{+\infty} x_i.P_{x_i}(x_i)dx_i$

The marginal probability density function of x_i: $P_{x_i}(x_i)$

Correlation matrix: R_x

Covariance matrix: C_x

$$R_x = C_x + \mu_x.\mu_x^{\ T}$$

Condition of noncorrelation:

n x n diagonal matrix is displayed as: $C_x = E\{(x-\mu_x).(x-\mu_x)^T\} = D$

Noiselessness (cleaning-whitening-) condition:

$$\mu_x = 0 \qquad R_x = C_x = I$$

Orthogonal transformation:

$$Y = T_x \qquad T^T.T = T.T^T = I$$

Multivariate Gaussian distribution:

$$P_x(x) = \frac{1}{(2\pi)^{n/2}(\det(x))^{1/2}}\exp(-\frac{1}{2}(x-\mu_x)^T.C_x^{\ -1}.(x-\mu_x))$$

Thus, the multidimensional vector structure is as in the figure **(Figure 1).**

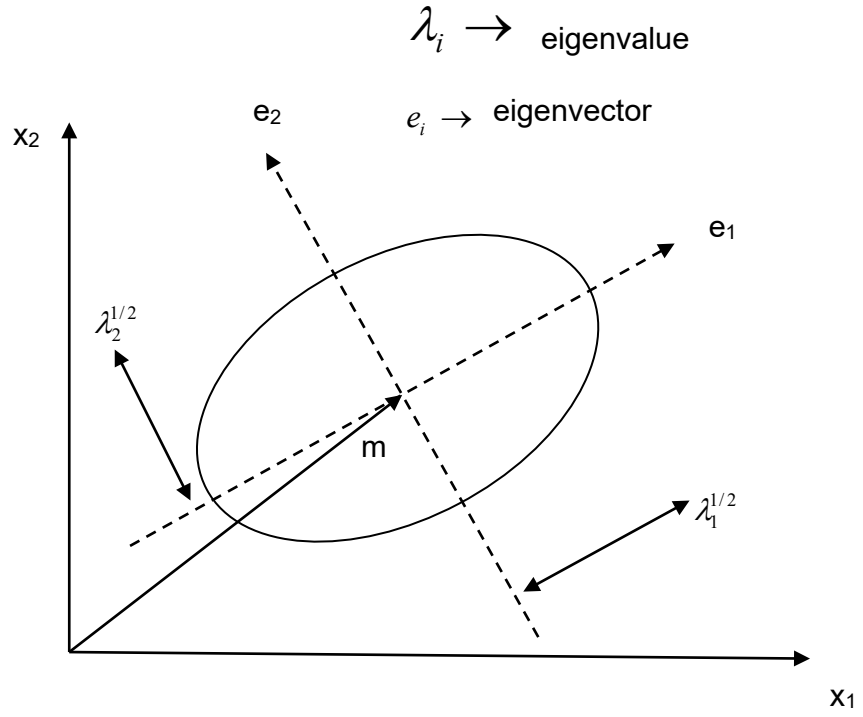

Figure 1: Multidimensional vector structure

m: number of variables , n:number of observations, t: time unit

Observed data : $x_i(t)$ i=1,…,m t=1,…,t

Component including : y_i ;

$$y_i(t) = \sum_j w_{ij} x_i(t) \qquad \text{i=1,…,n} \qquad \text{j=1,…,m}$$

To find w_{ij} coefficients;

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ . \\ . \\ . \\ . \\ . \\ y_n(t) \end{bmatrix} = W. \begin{bmatrix} x_1(t) \\ x_2(t) \\ . \\ . \\ . \\ . \\ . \\ x_n(t) \end{bmatrix}$$

Transformed y_i (t) components

Dimension Reduction: It is to be able to choose the W matrix with the smallest number of y_i components.

Independence: For data in a Gaussian distribution, unrelated components are also independent. But if the data does not show Gaussian distribution; This is where Independent Component Analysis comes in.

Unknown Source: The process of separating signals from different brain regions; It is the process of separating radio waves from mobile phone waves. The most typical example of this is the cocktail party problem **(Figure 2)** (9,10,11).

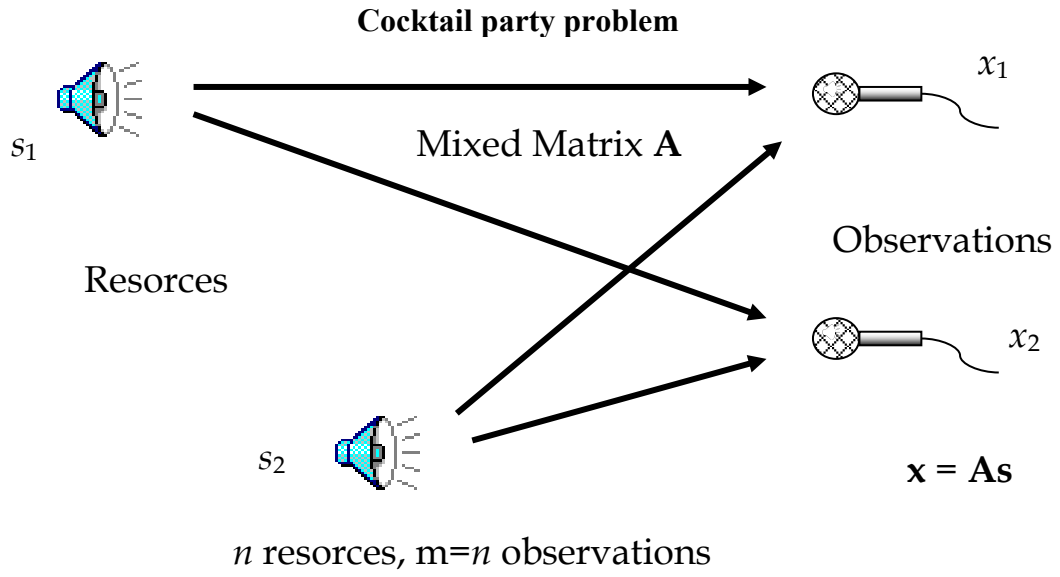

Figure 2: Cocktail party problem

Suppose there are 3 source signals;

Signals recorded at time t (observed): $x_1(t)$; $x_2(t)$ ve $x_3(t)$

real signals: let $s_1(t)$; $s_2(t)$ ve $s_3(t)$;

$$x_1(t) = a_{11}s_1(t) + a_{12}s_2(t) + a_{13}s_3(t)$$
$$x_2(t) = a_{21}s_1(t) + a_{22}s_2(t) + a_{23}s_3(t)$$
$$x_3(t) = a_{31}s_1(t) + a_{32}s_2(t) + a_{33}s_3(t)$$

We can use the W matrix to find the matrix composed of the mixture of a_{ij}.

Let the signals be as in the graph below **(Figure 3)**.

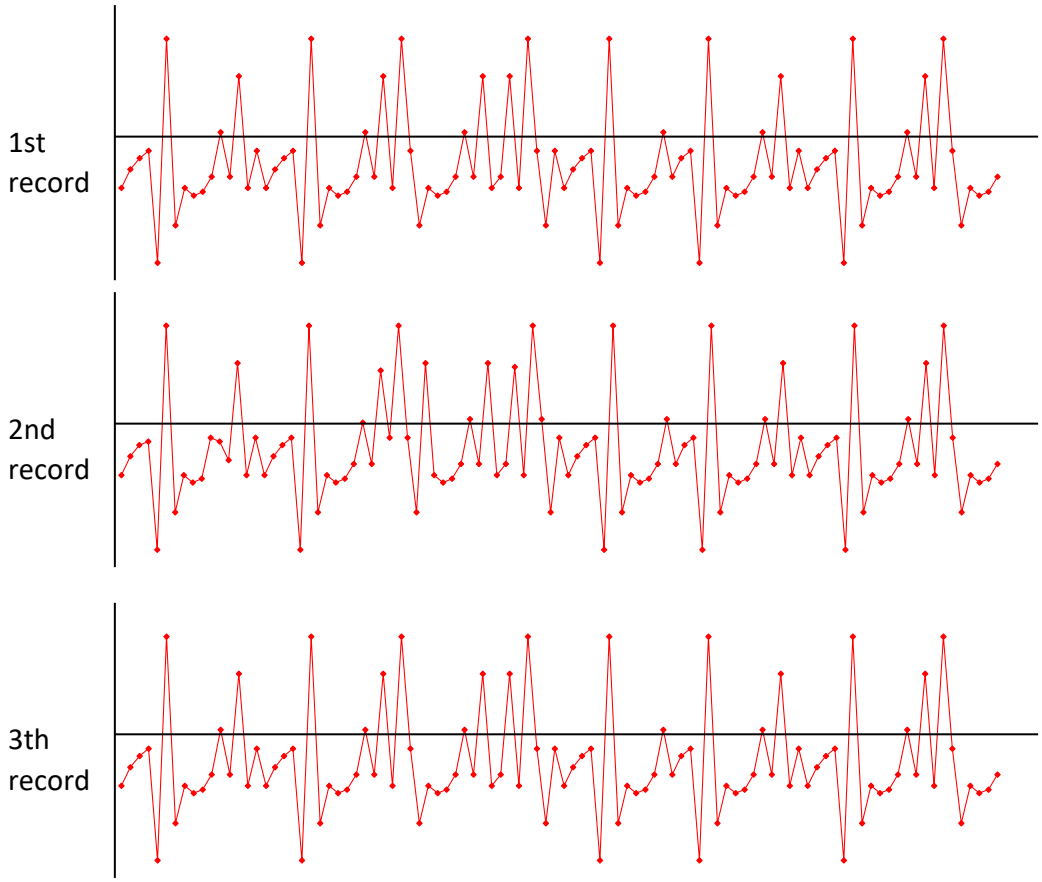

Figure 3: Three-source signals

Allocating resources based on independence, ie estimation of the coefficients of the Wij matrix;

$$y_1(t) = w_{11}x_1(t) + w_{12}x_2(t) + w_{13}x_3(t)$$
$$y_2(t) = w_{21}x_1(t) + w_{22}x_2(t) + w_{23}x_3(t)$$
$$y_3(t) = w_{31}x_1(t) + w_{32}x_2(t) + w_{33}x_3(t)$$

If y_1, y_2, y_3 become independent then x_1, x_2, x_3 will be equal to the original signals.

Thus, the signals separated by ICA; that is, the original signals (independent components) are separated as shown in the graphic below **(Figure 4)** (9,10,11).

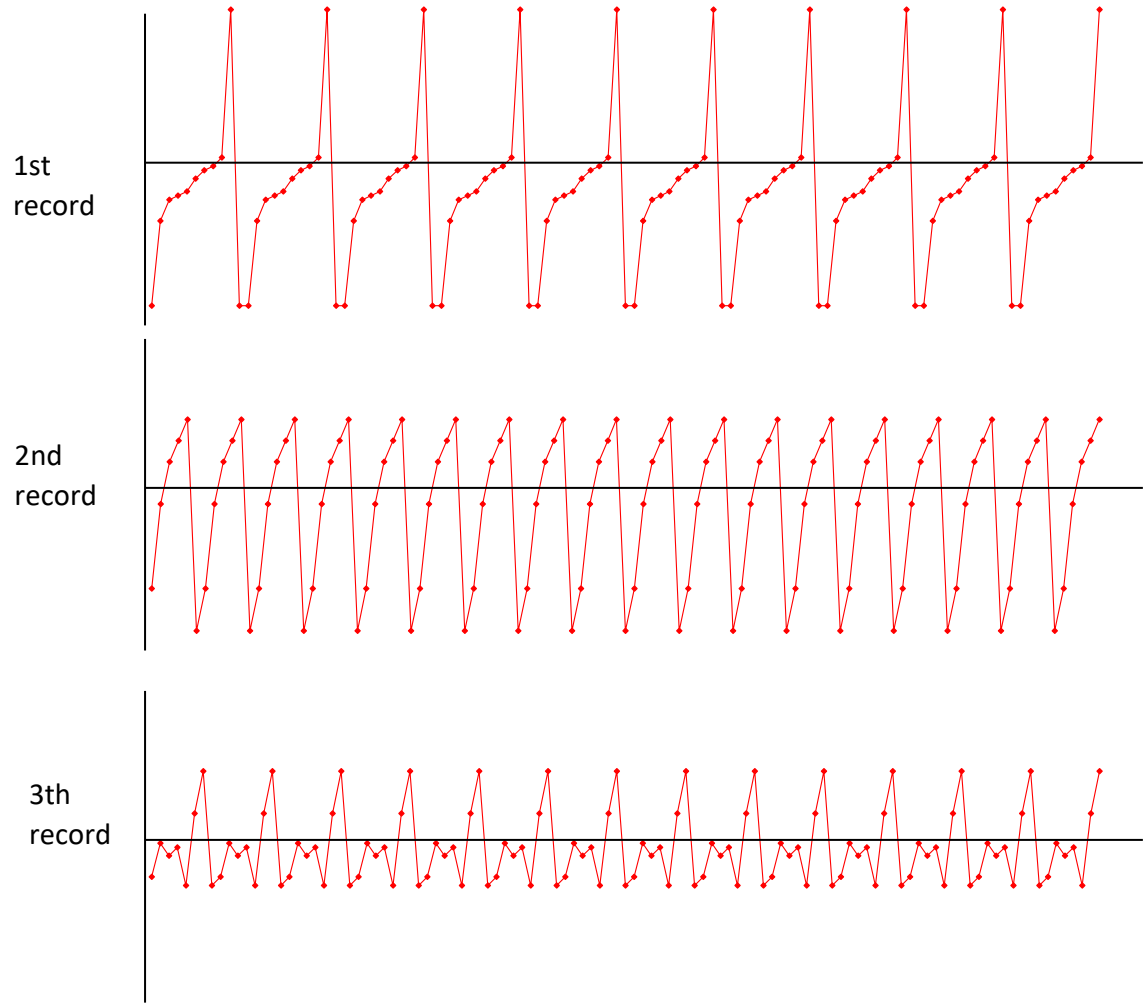

Figure 4: Separated signals

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_n(t) \end{bmatrix} = A. \begin{bmatrix} s_1(t) \\ s_2(t) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ s_n(t) \end{bmatrix}$$

Matrices that need to be estimated with ICA.

The observed matrix W is the inverse matrix of A. ($W=A^{-1}$) to find independent components, uncorrelatedness is not enough. Being independent is stronger than being unrelated. Principal component analysis or factor analysis cannot separate signals; however, they make it somewhat unrelated **(Figure 5)** (9,10,11).

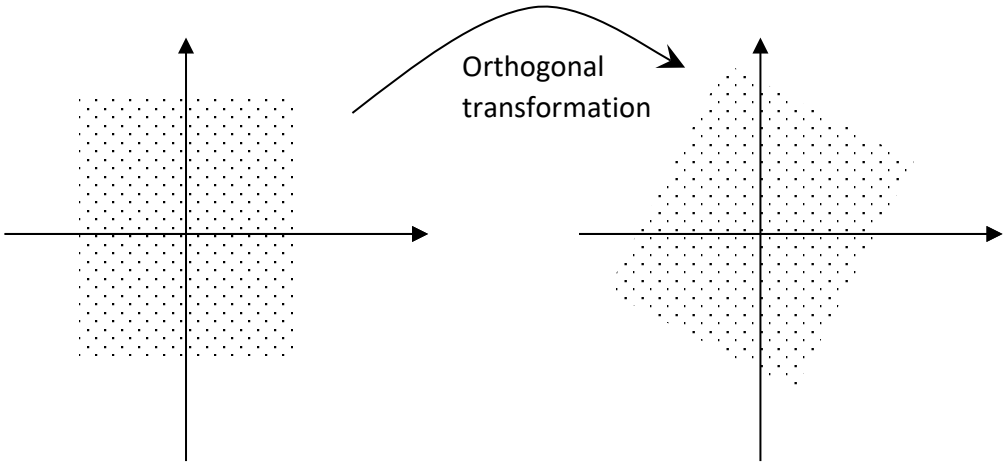

Figure 5 : Orthogonal Transformation

Although the figure on the right shows uncorrelatedness for the two independent components (S_1 and S_2) with uniform distribution in the figure, it still could not be separated.

It is necessary to look at whether independence is stronger than non-relationship in non-linear relationships. Let the nonlinear transformations of the independent components S_1 ve S_2 be $g(S_1)$ and $h(S_2)$. Covariance=0 is one of the principles of independent component analysis(9,10,11).

Another principle of ICA is the maximum non-gaussian situation. For the observed $y_i(t) = \sum_j w_{ij} x_j(t)$ linear combination, the non-gaussian state is maximized if any of them is equal to one of the independent components. If we had a real mixture of two or more components, it would approximate the gaussian distribution according to the central limit theorem(9,10,11).

2.2 HOW ARE INDEPENDENT COMPONENTS FOUND?

We can use a statistical model to describe the independent component analysis. Let's consider a series like $x_1,...,x_n$ with n linear mixture components;

$x_i = a_{j1}s_1 + a_{j2}s_2 + ... + a_{jn}s_n$ for all j values.

Now that we have dropped the time indicator t in the ICA model, we assume that each x_j is a random variable, such as the independent s_k component, rather than a real time signature. The

observed variables $x_j(t)$ are now a sample of this random variable. Without losing generalizability, we assume that both the mixed variables and the independent components have a mean of zero. If this is not true, observable variables (x_i) can always be centered by subtracting the sample mean. This makes the model zero-mean(9,10,11).

It is more convenient to use vector-matrix notation instead of sums in the previous equation. Let x denote any vector whose elements are mixtures of x1, ..., xn, and any vector with elements s1, ... ,sn as s. Let's denote the matrix whose elements are aij with A. Generally, lowercase letters in bold represent vectors, and uppercase letters represent matrices. All vectors are actually one-column matrices, and their transpose is one-row matrices. For the mixed pattern vector-matrix notation X=As above, we may sometimes need the columns of the matrix A. We also denote them with aj. The model in the form $x = \sum_{i=1}^{n} a_i s_i$ is a generative model. That is, it defines how the observed data is produced by mixing the si components. Independent components are latent variables. That is, they cannot be observed directly. It is also assumed that the mixing matrix is unknown. All observed is the x vector. We have to calculate both A and s using this. And this should be done under general assumptions as much as possible(9,10,11).

2.3 ASSUMPTIONS OF ICA

1- Statistical independence of the components: In fact, no more than this assumption is required, and this assumption is what makes the ICA strong. Statistical independence;

For $P(y_1, y_2, ..., y_3)$ the joint probability density function;

If $P(y_1, y_2, ..., y_3) = P_1(y_1).P_2(y_2).....P_n(y_n)$ each is said to be independent.

2- Independent components are in a non-Gaussian distribution. If it is known in advance that the distribution of the components is normal, the problem will simplify itself. Repetition of signals in a temporal structure requires a non-Gaussian distribution (9,10,11).

The most well-known criterion for whether or not it is in a Gaussian distribution is kurtosis. It is defined as:

$$\text{Kurt(y)} = E\{y^4\} - 3\left(E\{y^2\}\right)^2$$

The second criterion and one of the basic methods in ABB is to benefit from information theory. Non-Gaussian criteria will also cause some of the information about the distribution to be sacrificed. This can also be called negentropy. For this reason, information theory and entropy – negentropy criteria come into play (9,10,11).

3. MATERIALS AND METHODS

An EEG data showing no clinical signs was used to apply ICA. The main purpose; by revealing the more dominant components; It was to create a new subset of EEG signals using only these

components, as in the problem of "Separation of Unknown Source". Thus, Independent Component Analysis was applied to the signals obtained. Since it was not considered necessary for a thesis, an experimental case-control study was not carried out by repeatedly performing ICA on signals obtained from more than one person. The reason for this was that the findings obtained after ICA could be presented entirely as visual data. This meant n EEG graphs for n people. Finally, the results were visual results obtained from interfaces using MATLAB resources. Any statistical results; Although there are intense statistical formulas in the structure of the ICA, they could not be obtained.

In addition, this evaluation could be made with more complex analyzes, but since the purpose of this thesis is only ICA, these applications were left for later in order not to expand the size of the sample application unnecessarily.

For this reason, ICA was applied to a single EEG data, and the result was presented graphically and a special literature review was not made since the interpretation of the clinical issue fell into the field of neurophysiology.

Calculations; It was tested in the ICALAB and EEGLAB interfaces, which are the most frequently used all over the world and written for MATLAB.

4. FINDINGS

A section from the sample raw EEG data is shown below.

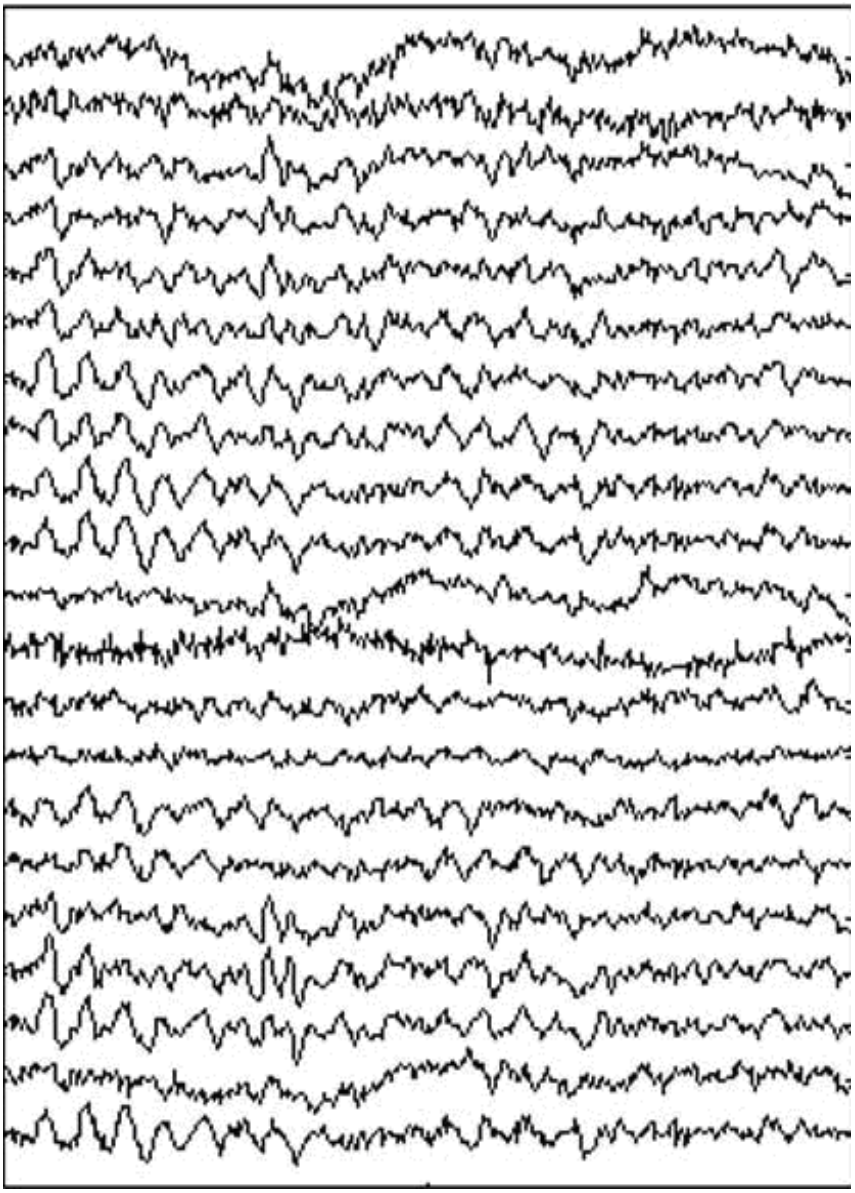

Figure 6 : Sample EEG Data Section

The seperated form of the EEG data, the section of which is given above, as a result of ICA.

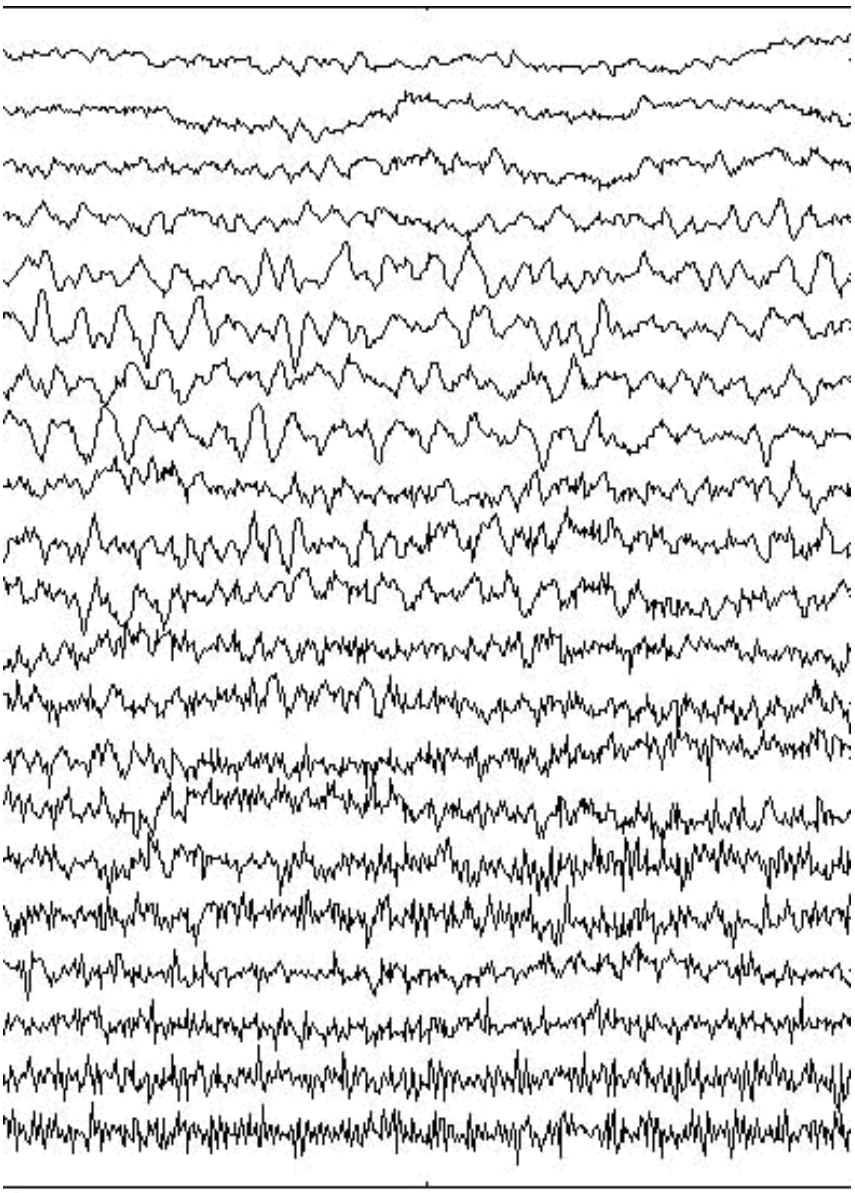

Figure 7: Seperated EEg Data Section

5. DISCUSSION AND CONCLUSION

Intuitively; if dealing with a disease that is very specific and is itself clinically complex; It is necessary to be more sensitive in obtaining some hidden components from the complex signals obtained from the EEG. Explaining whether the obtained components are related or unrelated should be investigated in further clinical studies. Each component obtained from the EEG does not necessarily mean that it is obtained from any part of the brain. Activities derived from such extra-brain sources are called artifacts. There are different sources of artifacts. For example, eye movement, blinking, body movements, muscle contractions, heartbeats, magnetic field changes created by moving objects in the EEG area, alternating current frequency in city

electricity are among these. Examining whether such components are segregated accurately and reliably is again based on highly sophisticated methods. ICA is a candidate to be a fast and reliable method for artifact separation.

A second application area for ICA, besides artifact resolution, is source localization. This issue is of great importance especially in the detection of epileptic and organopathological focus. Organopathological focuses include hemorrhages occurring in certain parts of the brain, vascular occlusions, and space-occupying lesions. Although many of these can be demonstrated directly by imaging techniques (MRI, etc.), organic pathology must reach a certain size. Electrophysiological methods can exhibit pathology even at a stage where imaging facilities are not available. As you know, detection of epileptic focus is based entirely on electrophysiological parameters. For this purpose, a number of algorithms and analysis methods are used. ICA may be one of them.

Clinical interpretations for the components obtained after ICA can be interpreted by neurologists and neurophysiologists outside of this thesis.

Footnote

This article is adapted from Ömer Uysal's doctoral thesis (26).

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